Proceedings of the Fifth World Congress on Theory of Machines and Mechanisms – 1979
Published by The American Society of Mechanical Engineers

DIAGRAM METHOD FOR TEACHING MECHANISMS AND MACHINES

Z. Levai, Professor
Budapest University of Technology, Hungary

The way emloyed in the diagram method reminds of making oscillograph tape. The graphs, built up by calculations and logical deduction, clearly and efficiently describe processes occurring in mechanisms. It can be used also for developing analysing abilities of students and for examinations as well.

Das in der Diagramm-methode angewandte Verfahren erinnert an die Herstellung des Oszcillografenbildes. Die auf Grund der Berechnungen und logischen Folgesungen aufgebaute Grafikone demonstrieren anschaulich und wirkungsvoll die Vorgänge, die sich in der Mechanismen abwickeln. Die Methode verwendbar auf die Entwicklung der Fertigkeit zur Analyse der Studenten und auch auf die Examinationen.

In education, often quite simple methods are efficient especially if it is consequently used. A simple idea will be presented likely of clearly, efficiently describing processes occurring in mechanisms. Another advantage of diagram method is to develop analyzing abilities of students by home practice. A third advantage consists in the easy and reliable testing of knowledge and analyzing abilities of students, for instance in examination. This simple method will be illustrated on a few examples rather than by lengthy explanations.

Let us take first a planetary gear train and present its motion conditions. Be this planetary gear train of P/P/N type, i.e. positive /external/ central gear plus positive planet gear plus negative /internal/ central gear. Be the relative velocity ratio Q=-0,5.

Any planetary gear train is known to be accessible to analysis by the theorem of relative velocity ratio:

$$Q = \frac{\omega_2 - \omega_3}{\omega_1 - \omega_3} \quad , \quad \text{where} \qquad /1/$$

ω_1 = angular velocity of the first central gear,
ω_2 = angular velocity of the second central gear,
ω_3 = angular velocity of the gear arm.

The equation 1 can written in form

$$Q\omega_1 - \omega_2 - /Q-1/\omega_3 = 0. \qquad /2/$$

If two of the velocities are given, the third one can be determined. This method is not illustrative and not vividly descreptive. In Europe, the Kutzbach grapfhic method is in general use, examining the angular velocity ratios by the vectors of momentary peripheral velocities. Unfortunately, this method is too static to present the velocity changes.

Our diagram method is essentially based on the graphic display of the analytic equation 2 in parametered form:

$$\begin{aligned} \omega_1 &= \omega_1/t/ \\ \omega_2 &= \omega_2/t/ \\ \omega_3 &= \omega_3/t/ \end{aligned} \qquad /3/$$

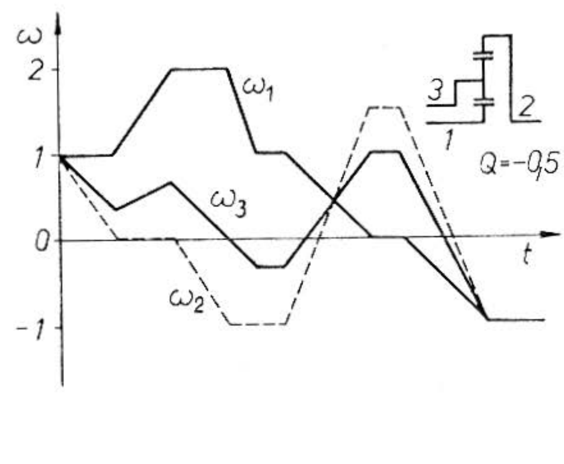

Fig.1.

Fig.1 shows the progress of all velocities. Two of them, namely ω_1 and ω_3, were prescribed by the teacher and ω_2 was to plot by the student /dashed line/. Of course, in applying the diagram for explanation, the dashed line will be traced in cooperation with the students, discussing the steps of solution.

The same problem may be put several time in an examination, since the in ital graphs are easy to modify.

The way employed in the diagram method reminds of making oscillograph tape. It has to be pointed out that in this method, time as a parameter is of no importence as a rule, the time scale may mostly be omitted in the diagram.

This example was a rather simple one, the application of diagram method could be seen a bit forced. It is more appropriate in cases where, in addition to displacements, also forces intervene.

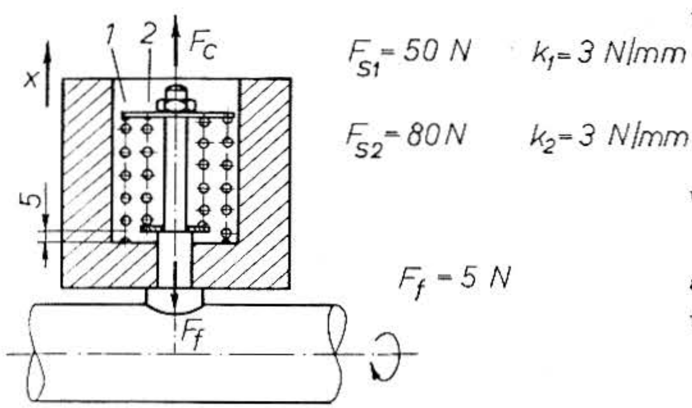

Fig.2.

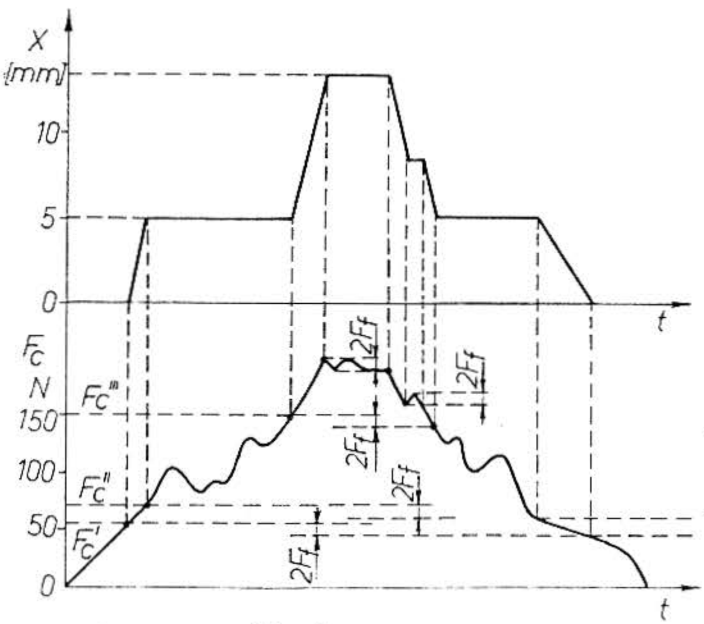

Fig.3.

Fig.2 shows the weight of a centrifugal regulator. Let the centrifugal force vary according to the graphin Fig.3 below. Centrifugal force may impose a displacement on the fly weight in direction x. The motion is counteracted by spring forces and friction. Spring data are shown in the figure. Be friction force F_f constant.

The problem is to determine the motion of the fly weight. It is correctly solved by performing the following calculation:

a/ $\quad x = 0, \qquad$ /4/

until $\quad F_c < F_{s1} \pm F_f.$

b/

$$x = \frac{F_c - F_{s1}}{k_1}, \qquad /5/$$

until

$$F_{s1} \pm F_f < F_c < F_{s1} + k_1 a \pm F_f.$$

c/ $\quad x = a, \qquad$ /6/

until $\quad F_{s1} + k_1 a \pm F_f < F_c < F_{s1} + F_{s2} + k_1 a \pm F_f$

d/

$$x = \frac{F_c + k_1 a - F_{s1} - F_{s2}}{k_1 + k_2} \qquad /7/$$

until $\quad F_c > F_{s1} + F_{s2} + k_1 a \pm F_f.$

Of course, students have to know when F_f is affected by a positive or a negative sign, as well as that if F_c oscillates, there is only displacament if $\Delta F_c > F_f$.

The correct solution is that according to the line in Fig.3.

Naturally, this problem may have several reuses in examination. The graph of $n = n/t/$ may be put in different manners. But the question may be reverted: the graphs $x = x/t/$ and $n = n/t/$ are given and the pertaining spring date / $F_s = ?$, $k = ?$ / asked for.

The third problem will involve, in eddition to displacement and force, also the pressure.

The mechanism is that shown in Fig.4.

The problem is to determine the relationship between p_1 and p_2 and to show the motion of piston E. In Fig.5 the graph of p_1 has been plotted by the teacher in advance. For the sake of simplicity, this process will be statically considered: pressure p_1 is changed in infinitely small steps and p_2 and x are awaited to be stabilized after each step.

The solution procedure is the following.

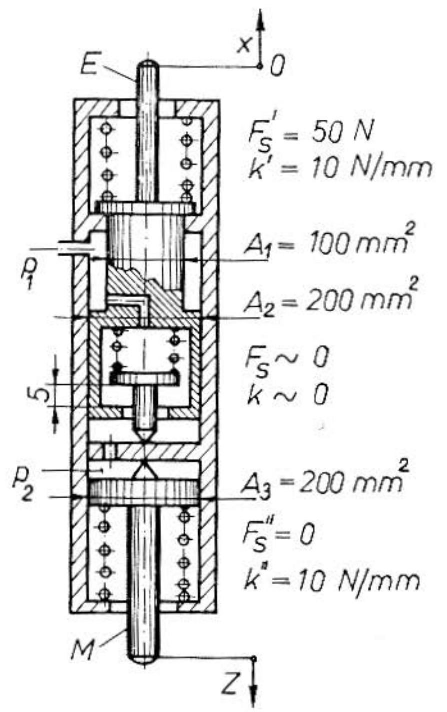

Fig. 4.

Piston E is immobile /x=0/ hence $p_1 = p_1$

until $\quad p_1 < \frac{F_{s1}}{A_1}$

After the start of piston E:

$$x = \frac{A_1}{k_1} p_1 - \frac{F_{s1}}{k_1} \qquad /8/$$

until $\quad \frac{F_{s1}}{A_1} < p_1 < \frac{F_{s1} + k_{1a}}{A_1}$

In this case

$x = 4p_1 - 2$ until $0{,}5 < p_1 < 1.$

Meanwhile, with the valve open:

$$p_2 = p_1. \qquad /9/$$

After the valve has closed, that is when

$p_1 > \frac{F_{s1} + s1^a}{A_1}$, i.e. $\quad p_1 > 1,$

piston E allways returns to $x = a$ after each increment of p_1. This time equilibrium of forces acting on piston E:

$$p_2 A_2 - /A_2 - A_1/ p_1 - F_{s1} - k_1 x = 0 \qquad /10/$$

hence

$$p_2 = \frac{A_2 - A_1}{A_2} p_1 + \frac{F_{s1} + k_1}{A_2} = 0{,}5\, p_1 + 0{,}5. \qquad /11/$$

If the trend of variation of p_2 turns back, i.e. decrease occurs instead of increase, it calls for taking also piston M into consideration.

General equilibrium of forces on piston M:

$$p_2 A_3 - F_{s2} - k_2\, y = 0. \qquad /12/$$

In the end position:

$$p_2^{max} A_3 - F_{s2} - k_2\, y^{max} = 0 \qquad /13/$$

Incompressibility of fluids involves:

$$/x-a/\ A_2 - /y^{max} - y/\ A_3 = 0. \qquad /14/$$

Eqs 12, 14, 15, and 16 yield expressions for both $p_2 = p_2/p_1/$ and $x = x\,/p_1/$ such as:

$$p_2 = 0{,}25\, p_1 + 0{,}5\, p_2^{max} + 0{,}25, \qquad /15/$$

and

$$x = 10\, p_2^{max} - 5\, p_1. \qquad /16/$$

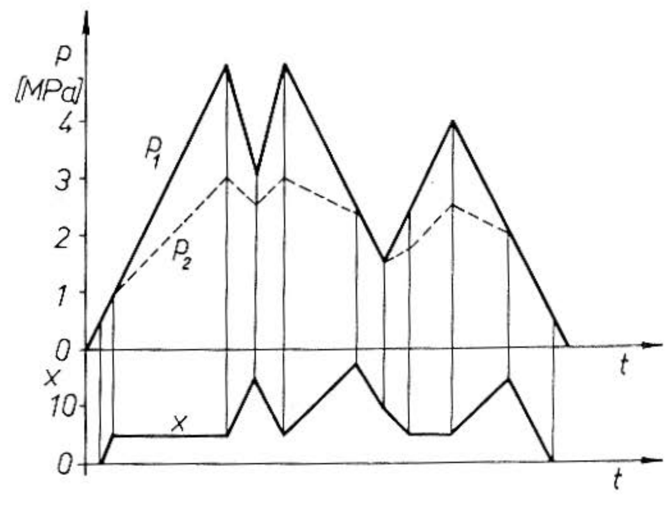

Fig.5.

Students have to be aware that Eqs 15 and 16 hold until p_2 drops to the momentary value of p_1, i.e. until

$$p_2^{max} > p_2 > \frac{2}{3}\, p_2^{max} - 0{,}33.$$

In this range, validity of Eqs 14 and 15 is independent of the trend of variation of the p_1 value.

Of course, below this range Eqs 8 and 9, above it Eqs 11 and x = a will hold.

The correct solution is that in dashed line in Fig.5. Of course, also this problem may be put in an infinity of variations.

Our last problem will concern a complex mechanism. Although its operation is accessible to mathematic analysis, here the graphic method will be applied only to check the sense or abilities for mechanism analysis. No concrete data will be given for the problem, neither will the students be asked to plot graphes with exact values. The character of the graphes is of interest. Dynamic opproach to the phenomenon may be asked for, that is to realize continous variations of displacements and pressures, rather than variations in infinitely small steps.

In Fig.6 the motion of point A was determined by the teacher while the graphs characterizing both the motion of points B, C, D and variations of pressures p_1, p_2 were to be ploted by the students.

In such a dynamic problem it matters if the student realized times Δt while points B, C and D are still in motion, although point A has already stopped.

No more explanation of the diagram method seems us to be necessary, it being fully clear after the examples. This method has been in use for more than a decade by us, a lot of problems have been developed. There are some needing only ten minutes to be solved, while others require more than an hour. Students consider this method as useful and interesting.

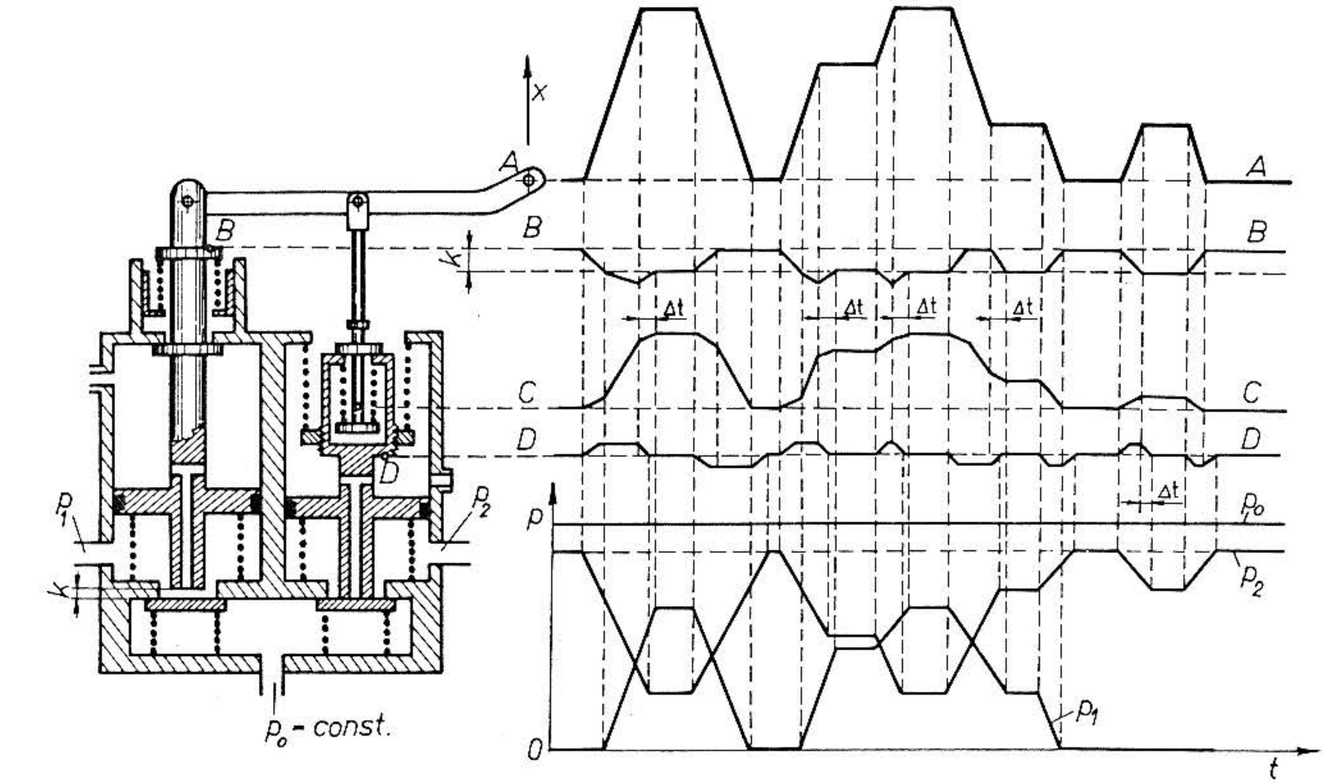

Fig. 6.